Powers of Ten - Introduction

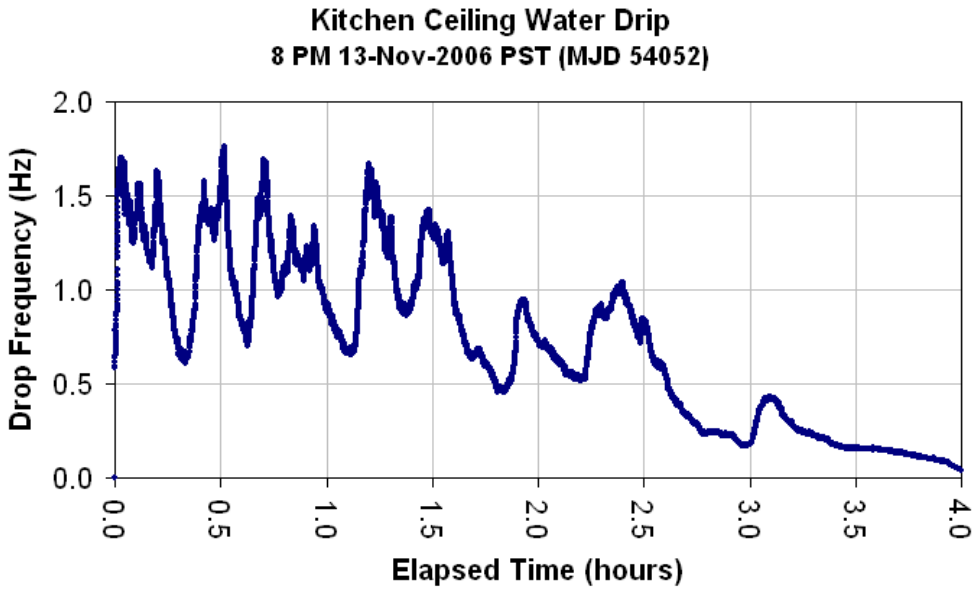
- A quick overview of clock accuracy
- What clocks keep poor time?
- What clocks keep best time?
- And many in between…

Note: this set of slides were 'chapter 5' of my presentation on Project GRE²AT at the PTTI conference in December 2006.

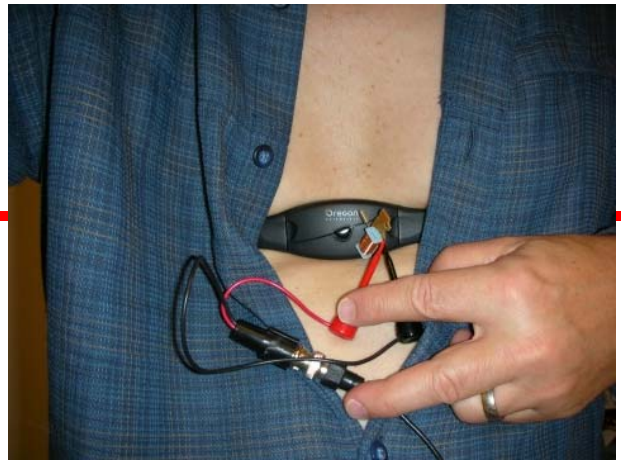
Tom Van Baak
tvb@LeapSecond.com
www.LeapSecond.com

10^{-0} drip, drip

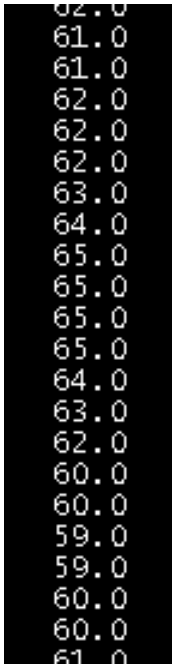
- Leak in ceiling
- 0.57 s … 9.9 s
- 1.7 Hz … 0.1 Hz



10^{-1} heart beat

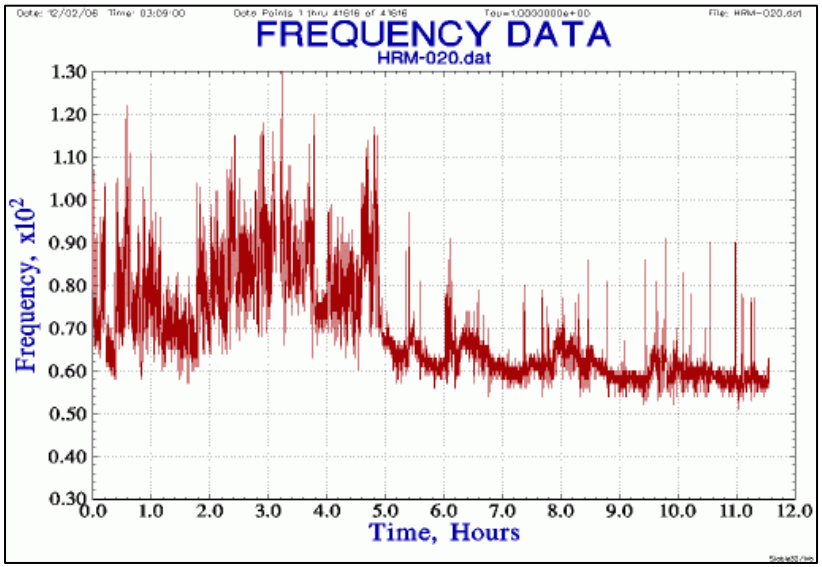


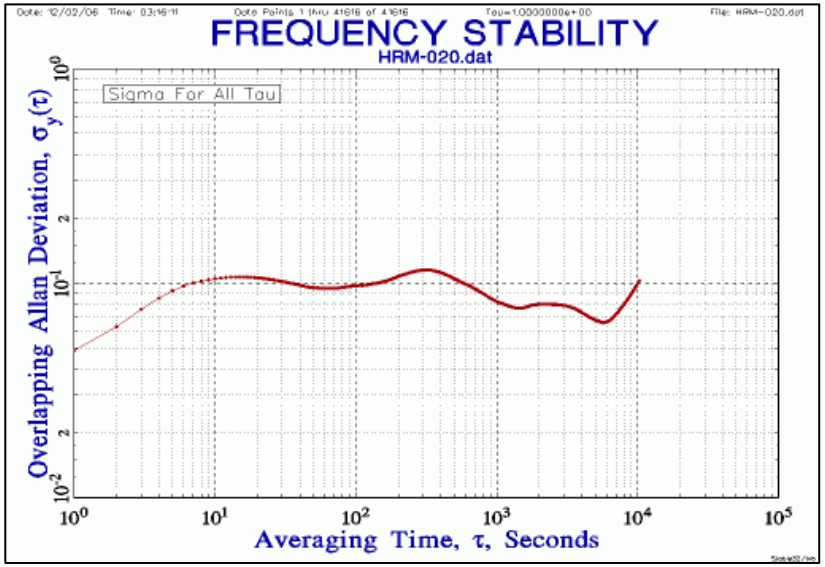
- 10^{-1}, 0.1, 10%
- The original '1 PPS'
- Sometimes 2x, even 3x
- Much higher stability at night
- < 10% accuracy possible



10^{-1} heart beat

- 12 h frequency plot (evening/night)
- ADEV floor is 10^{-1} from 10^{1} to 10^{4} s!
- (is this OK?)



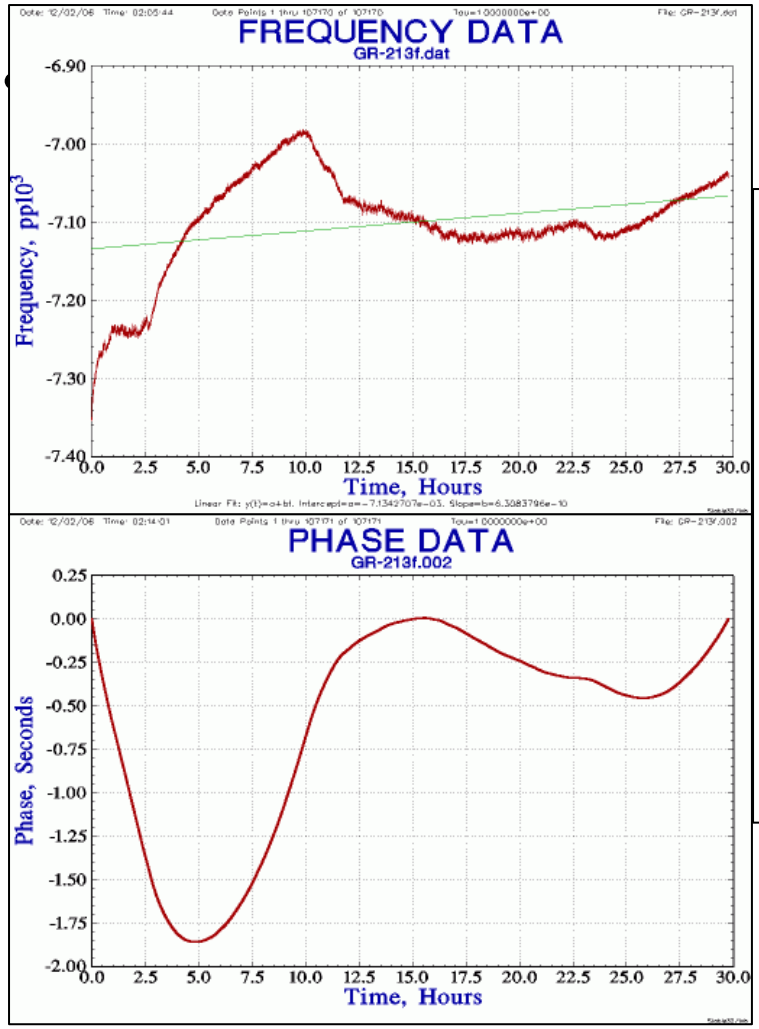


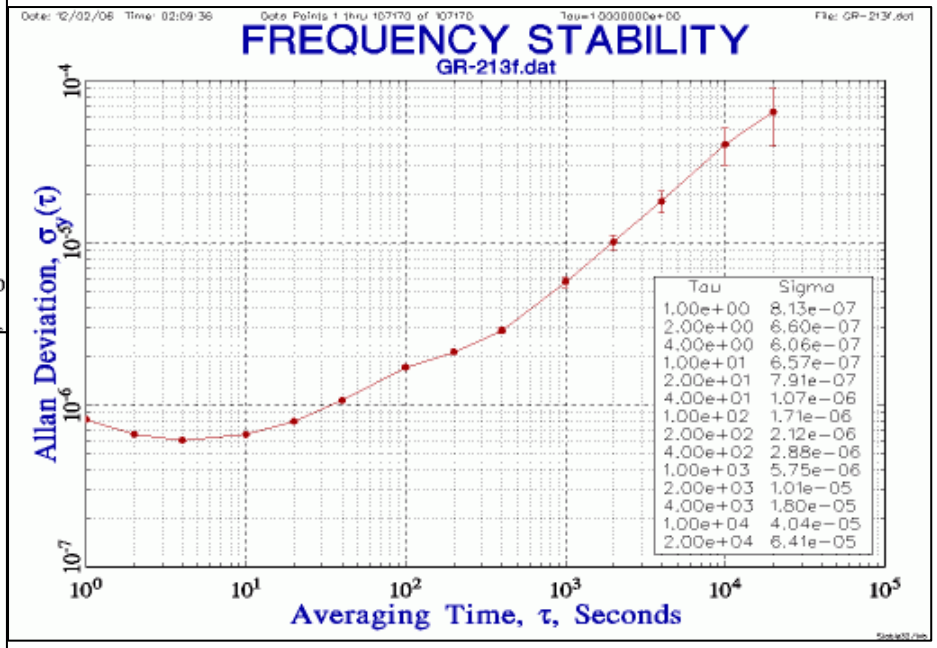
10^{-2} tuning fork oscillator

- 0.01, 1%
- General Radio Type 213 Audio Oscillator
- 1 'kc'; f = ~992.8 Hz
- ±1.3 mHz (60 x 1 s)
- Accuracy < 1%
- Count those 9's
- ADEV is 10^{-6}


```
992.897,388,71 Hz
992.896,598,37 Hz
992.896,556,22 Hz
992.896,560,05 Hz
992.897,374,78 Hz
N       : 60
STD DEV: 0.001,387,672 Hz
MEAN    : 992.898,857,676 Hz
MAX     : 992.901,768,32  Hz
MIN     : 992.896,168,74  Hz
992.898,234,03  Hz
992.898,247,28  Hz
992.897,293,73  Hz
```

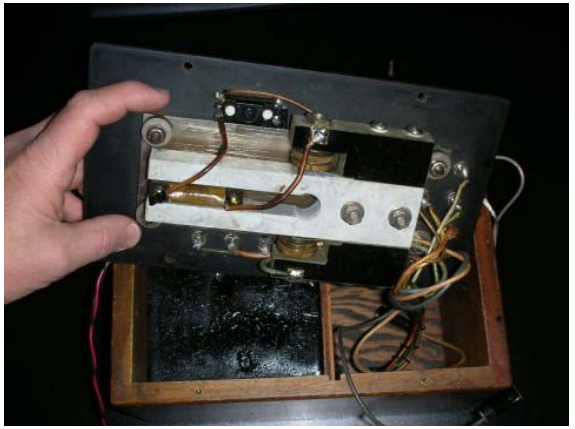
10^{-2} tuning fork oscillator





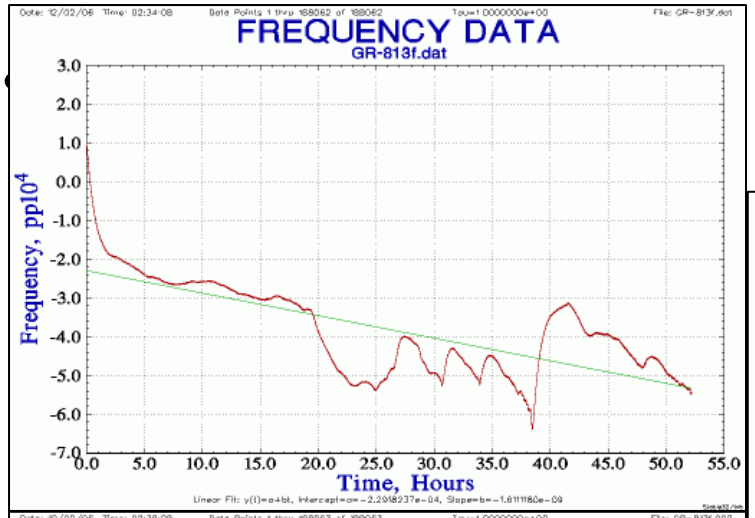
10^{-3} precision tuning fork

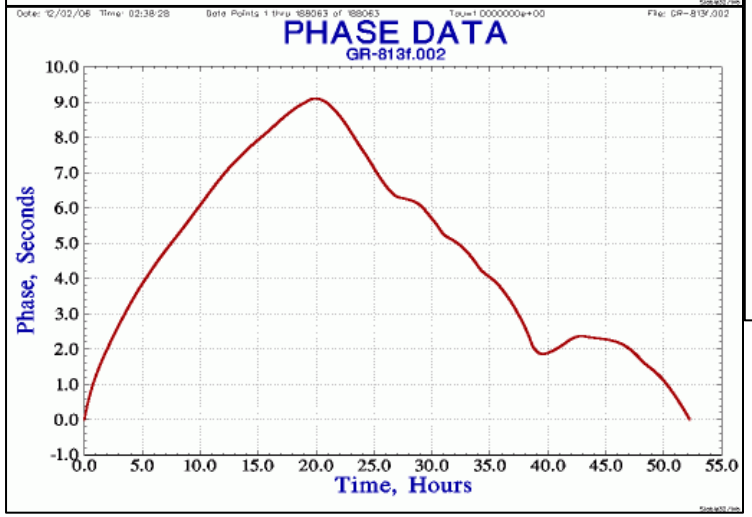
- 0.001, 0.1%, 1 ms/s
- General Radio Type 813
- 1 'kc' tuning fork
- f = ~999.4 Hz
- ±400 μHz (60 x 1 s)
- Accuracy < 0.1%
- ADEV is 10^{-7}

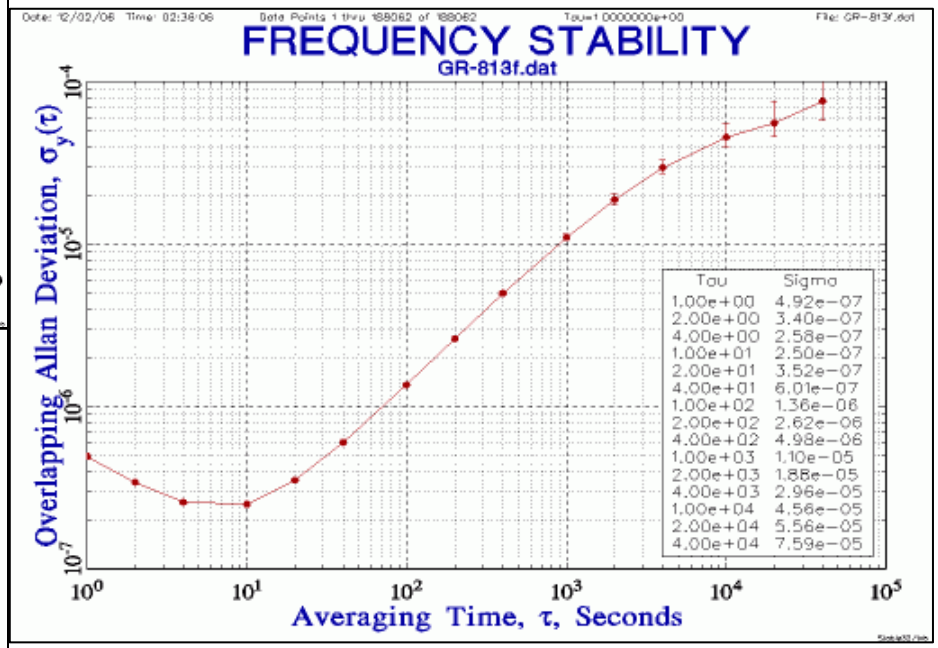


```
999.463,932,59  Hz
999.464,159,16  Hz
999.465,063,84  Hz
999.463,826,22  Hz
999.464,577,00  Hz
N       : 60
STD DEV: 478.778 uHz
MEAN    : 999.464,134,273 Hz
MAX     : 999.465,477,73  Hz
MIN     : 999.463,290,13  Hz
999.464,657,58  Hz
999.464,554,46  Hz
```

10^{-3} precision tuning fork



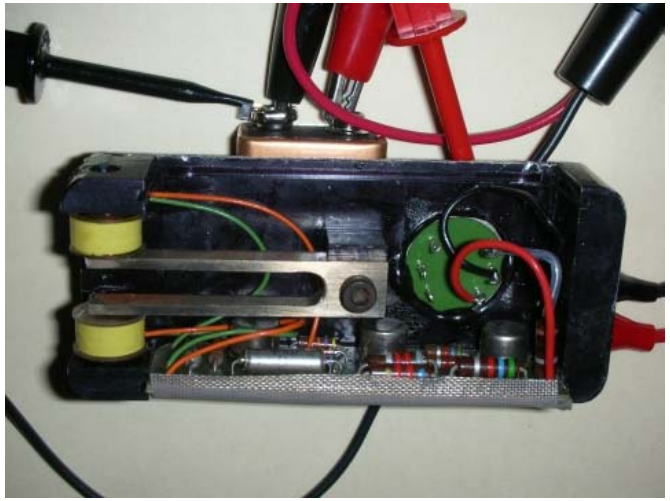




10^{-4} mechanical oscillator

- 0.01%, 100 ppm
- Mechanical oscillator
- "Four 9's"

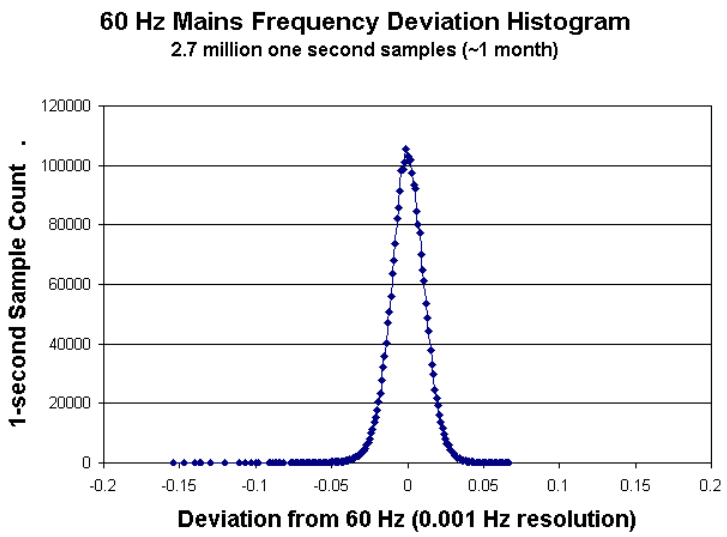
```
999.907,211,67  Hz
999.907,250,33  Hz
999.907,273,16  Hz
999.907,311,01  Hz
999.907,250,27  Hz
999.907,345,09  Hz
N       : 60
STD DEV: 151.812 uHz
MEAN    : 999.907,159,334 Hz
MAX     : 999.907,404,05  Hz
MIN     : 999.906,840,54  Hz
999.907,392,20  Hz
999.907,415,25  Hz
999.907,354,85  Hz
```



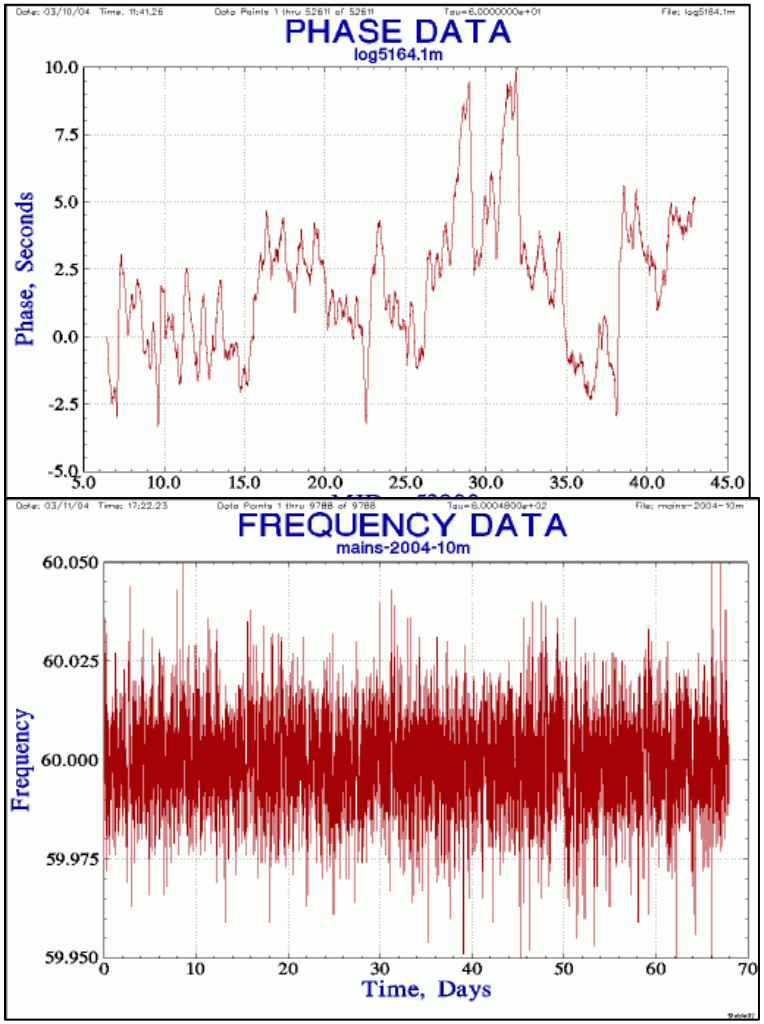
10^{-5} mains

- 0.001%, 10 ppm
- 60± Hz

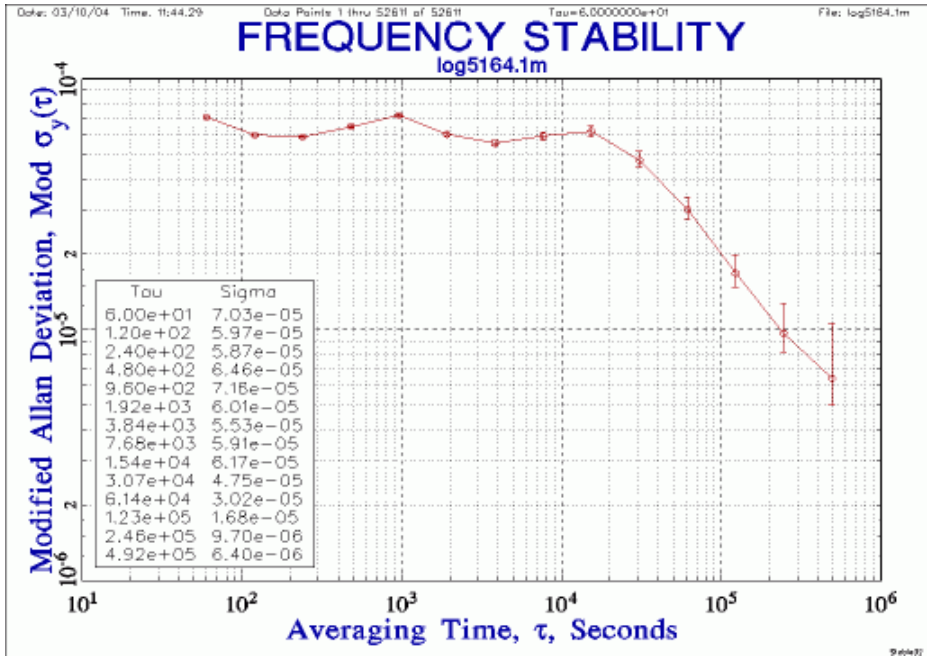
```
60.003,640,120,5  Hz
60.009,491,393,8  Hz
60.000,431,181,6  Hz
59.992,198,219,9  Hz
59.987,371,509,5  Hz
59.993,148,200,6  Hz
59.999,032,462,5  Hz
59.985,892,634,1  Hz
59.995,727,396,2  Hz
N       : 36
STD DEV: 0.006,765,596,40 Hz
MEAN    : 59.999,554,563,23 Hz
MAX     : 60.010,390,980,5  Hz
MIN     : 59.985,892,634,1  Hz
59.996,011,518,6  Hz
```



10^{-5} mains

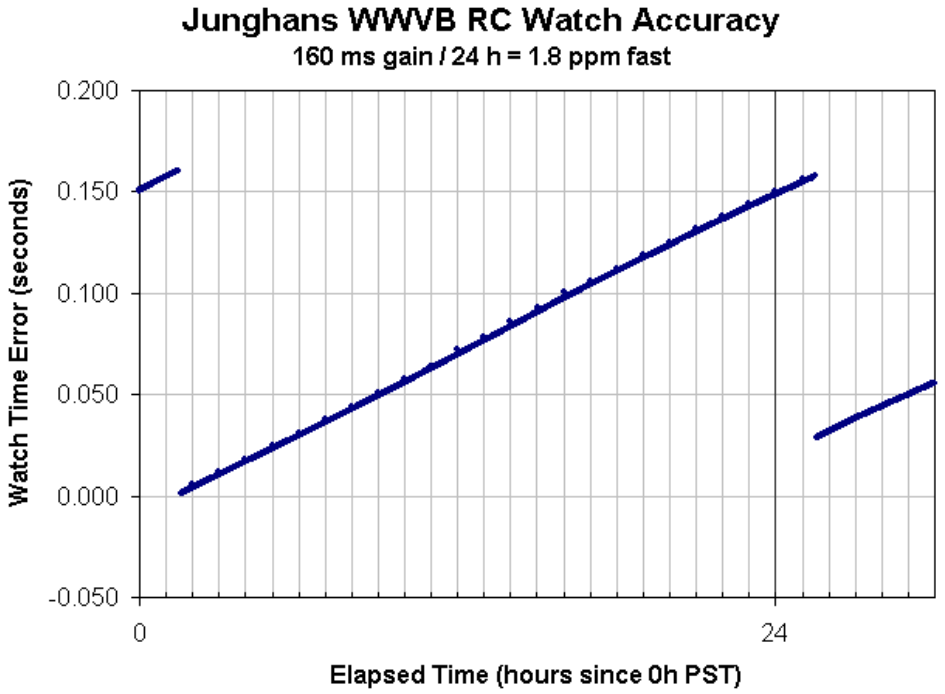

PHASE DATA
log5164.1m
Phase, Seconds
10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0

FREQUENCY DATA
mains-2004-10m
Frequency
60.050
60.025
60.000
59.975
59.950
0
10
20
30
40
50
60
70
Time, Days


FREQUENCY STABILITY
log5164.1m
Modified Allan Deviation, Mod $\sigma_y(\tau)$
Tau Sigma
6.00e+01 7.03e-05
1.20e+02 5.97e-05
2.40e+02 5.87e-05
4.80e+02 6.46e-05
9.60e+02 7.16e-05
1.92e+03 6.01e-05
3.84e+03 5.53e-05
7.68e+03 5.91e-05
1.54e+04 6.17e-05
3.07e+04 4.75e-05
6.14e+04 3.02e-05
1.23e+05 1.68e-05
2.46e+05 9.70e-06
4.92e+05 6.40e-06
Averaging Time, τ, Seconds

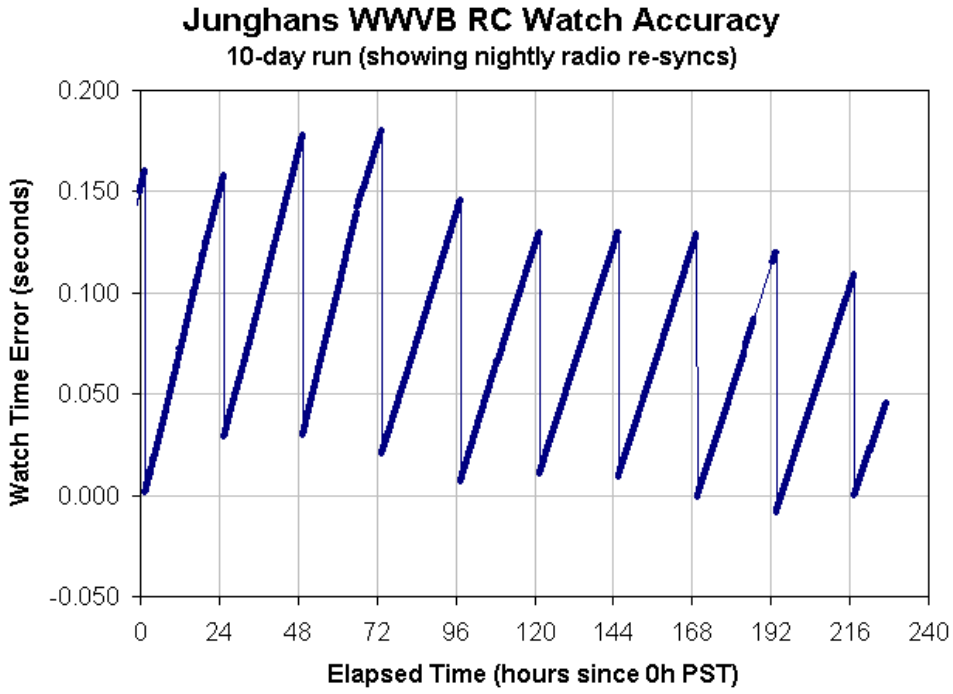
10^{-6} quartz watch (RC)

- 0.0001%, 1 PPM, 1 μs/s
- +160 ms/d = +1.85 ppm



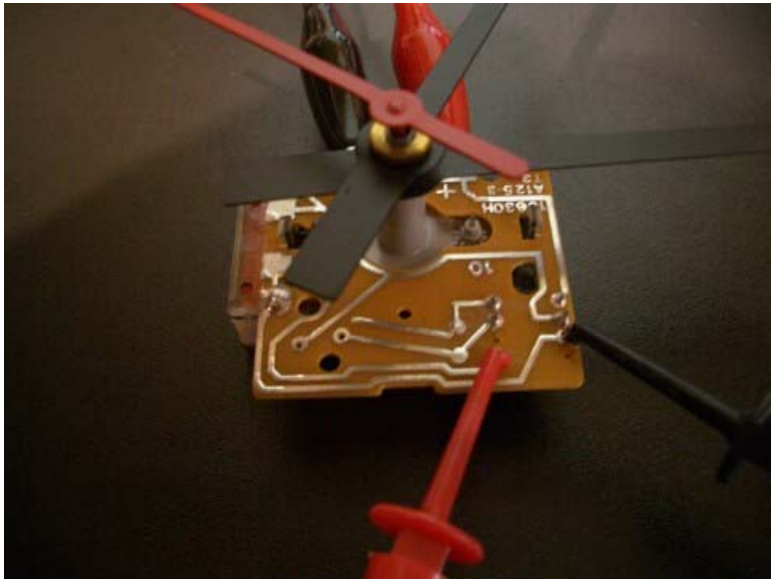
10^{-6} quartz watch (RC)

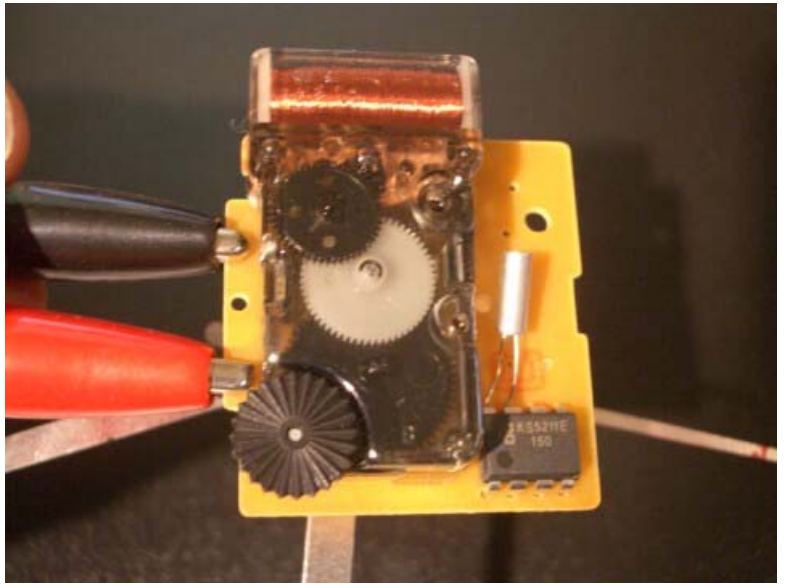
- Nightly WWVB radio sync (60 kHz)
- Look closely at 01:30 AM PST
- +1h +30m +15s
- Plot of 9 days
- Rate variations
- Sync variations



Aside: Quartz Wall Clock

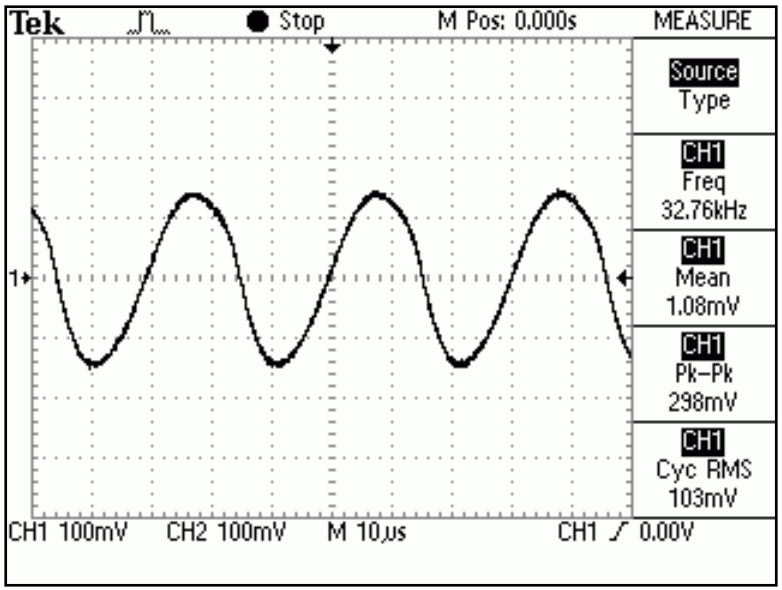
- Quartz crystal and divider/driver IC
- Stepper motor (180° per step)

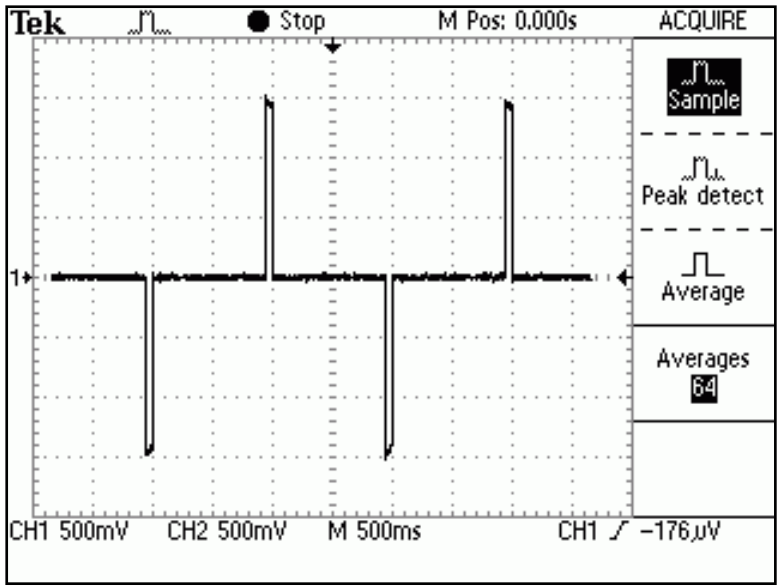




Quartz Wall Clock

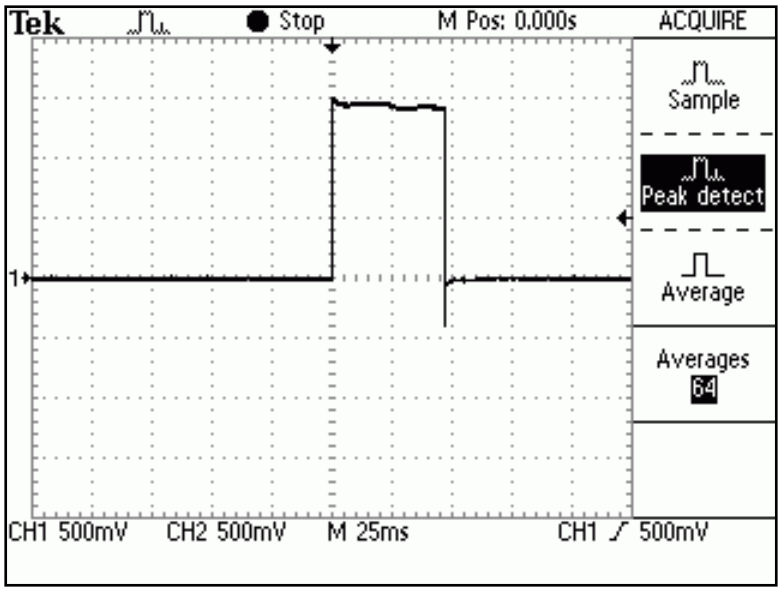
- 32 kHz oscillator
- 1 Hz stepper

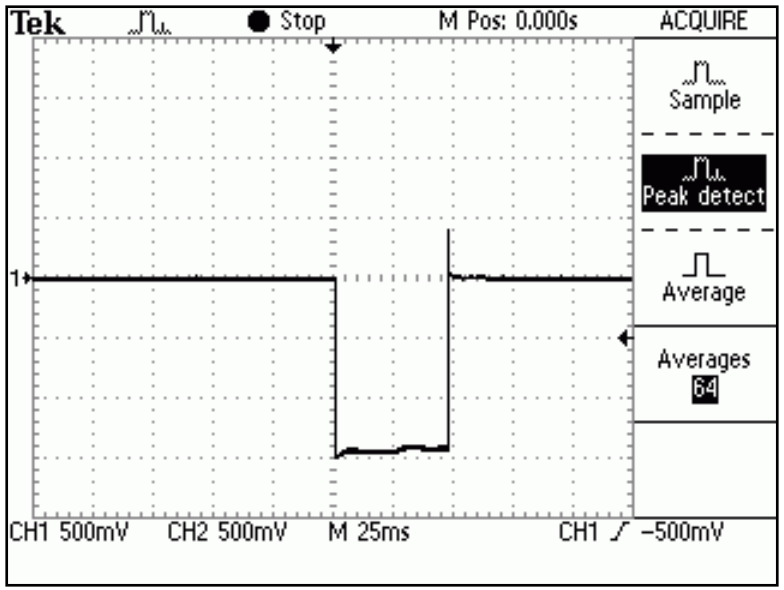




Quartz Wall Clock

- Polarity alternates
- Pulse size: 1.5 V x 50 ms





Quartz Wall Clock

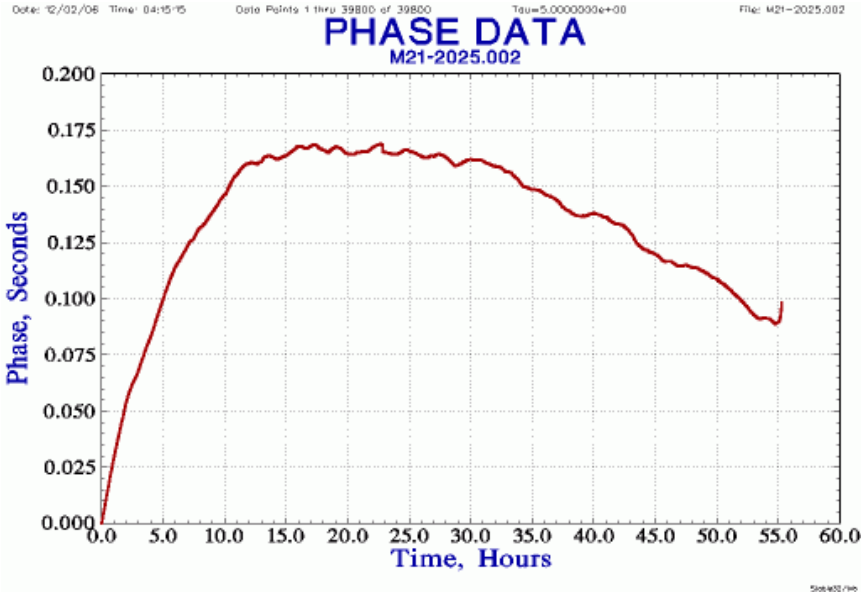
- Coil current: 1.5 V / 500 Ω = 3 mA
- Oscillator current: <1 μA
- Pulse power (V x A): 4.5 mW
- Pulse width: 50 ms
- Clock Energy (P x T): 4.5 mW x 50 ms = 225 μJ = 60 pico kWh
- AA battery (2850 mAh) = ~2 years

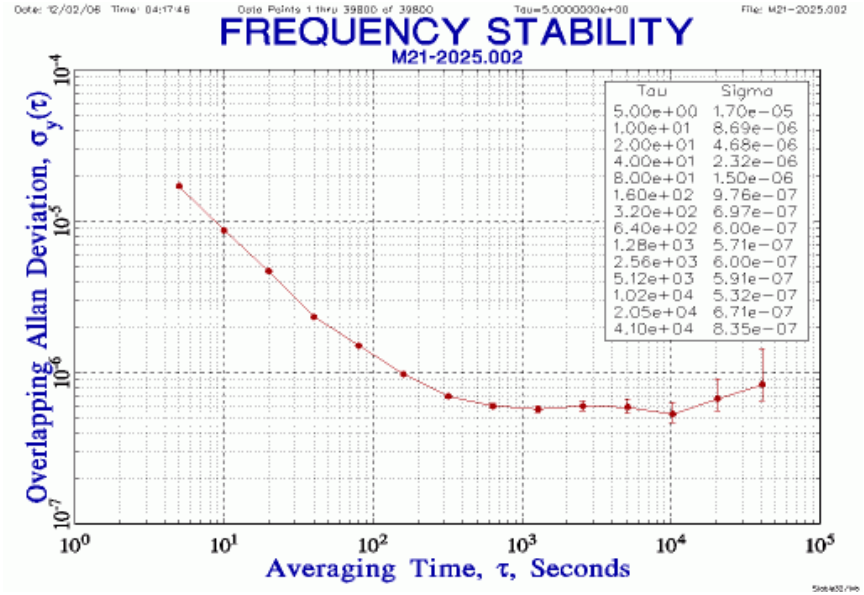
10^{-7} chronometer

- 0.1 ppm
- Rated $\frac{1}{4}$ sec/day deviation

10^{-7} chronometer

- ~55 hour runtime
- 200 ms phase residuals
- ADEV 6×10^{-7}





10^{-7} chronometer

- From 1940's USN manual…
- Phase
  - Dial error
- Frequency
  - Daily rate
- Drift
  - Deviation in rate

COMPUTATION OF RATE

Date	Dial Error + = Fast - = Slow		Daily Rate + = Gain - = Loss	Mean Deviation in Daily Rate	Remarks
	Min	Sec			
Oct 1948					
3	+0	2			Started + Set
4	+0	2½	+ ½		
5	+0	2½	0	¼	
6	+0	3	+ ½	¼	
7	+0	3	0	¼	
8	+0	3½	+ ½	¼	
9	—	—	—	—	Not wound
10	+0	4	+ ¼	—	2 day avg.

(Mean daily rate = +1/4 second)

In Table I, there will be noted a column headed "Mean Deviation in Daily Rate." The

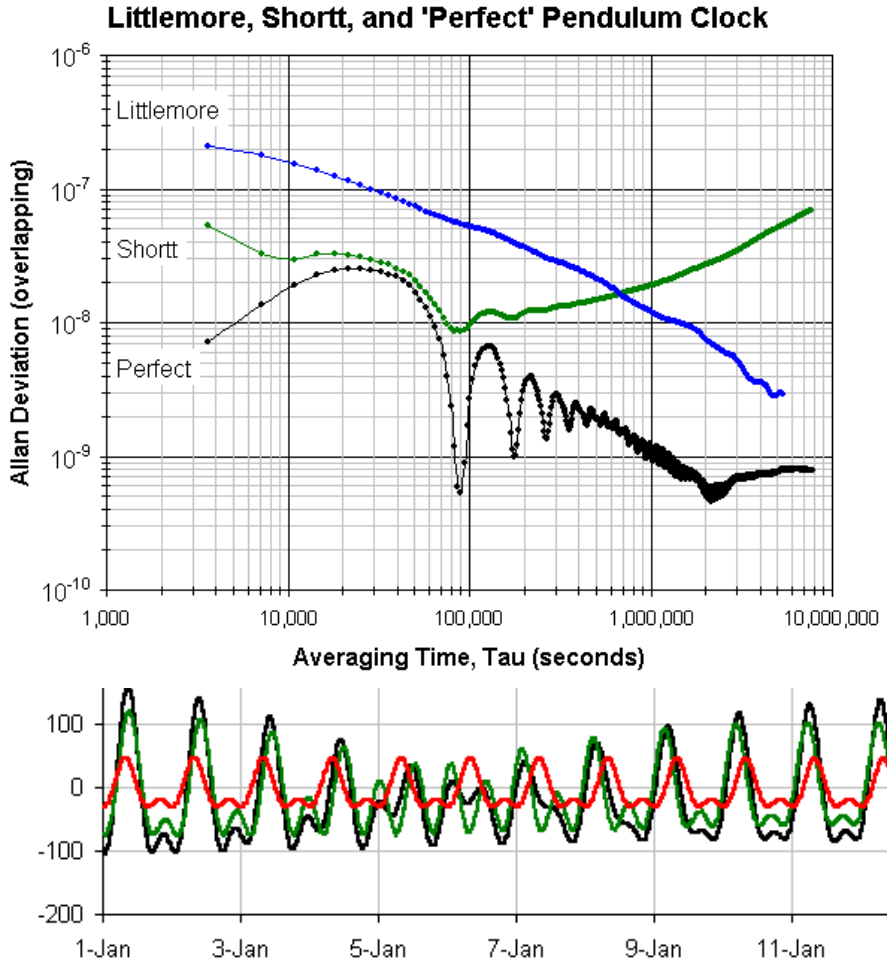
10^{-8} pendulum clock

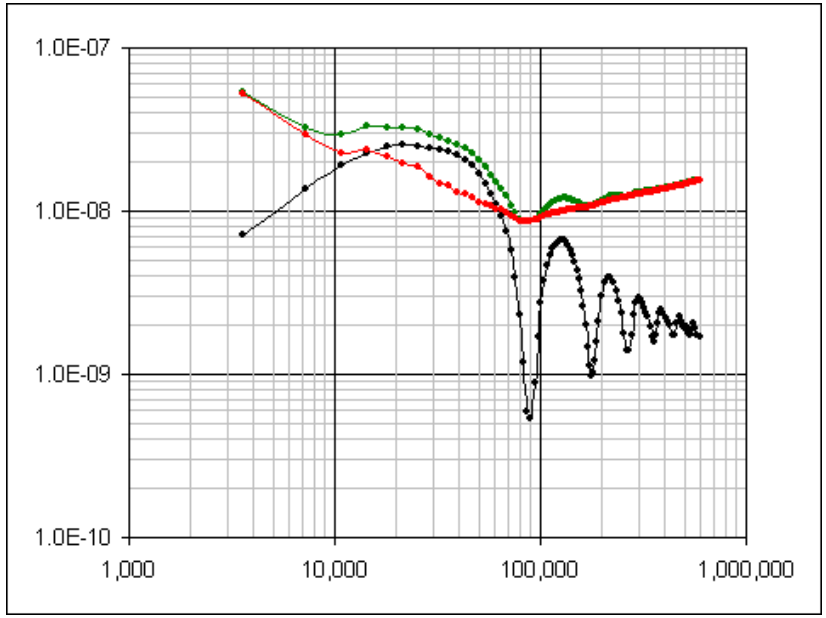
- 0.01 ppm, 10 ppb
  10 ns/s, 864 μs/d
- Shortt,
  Fedchenko,
  Riefler,
  'Littlemore'

10^{-8} pendulum clock

- Amazing astronomical pendulum clocks
- Several centuries of understanding and perfection. Limitations addressed:
- Temperature, humidity, mass, friction, metallurgy, escapement, master/slave, vacuum, isochronous suspension, etc.
- When all factors solved, the best pendulum clock is just a good gravimeter

10^{-8} pendulum clock

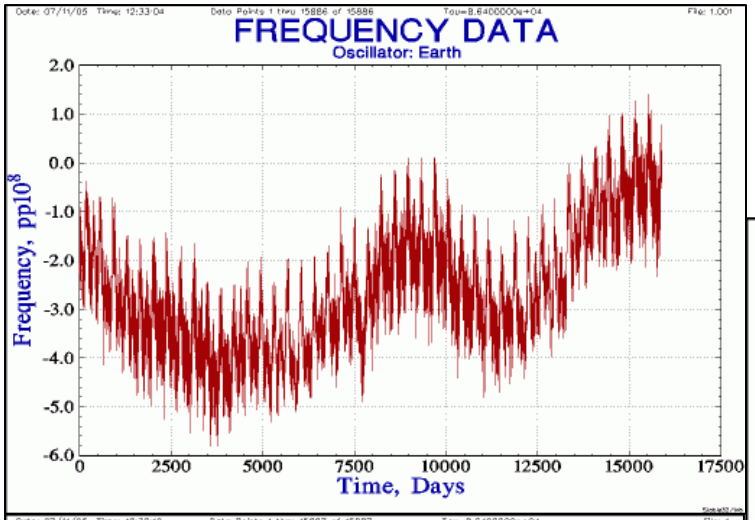


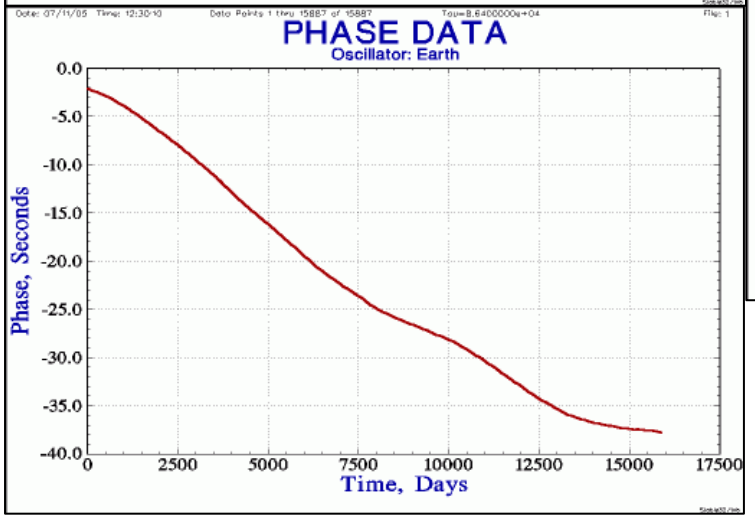


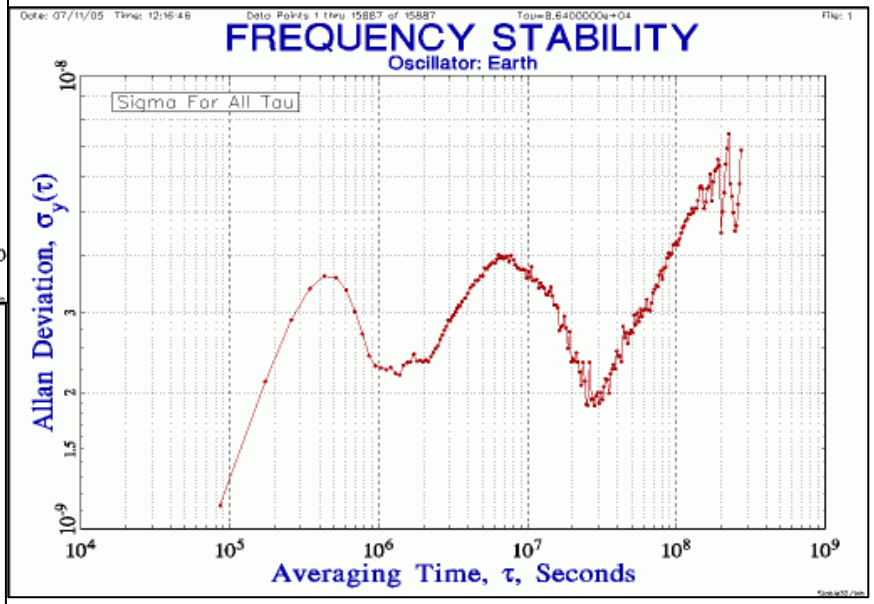
10^{-9} earth

- 0.001 ppm
- Slow by ~2 ms per day
- Also somewhat irregular
- ADEV 10^{-8} ~ 10^{-9}
- Limited by core, weather, climate
- Also lunar/solar tides

10^{-9} earth


FREQUENCY DATA
Oscillator: Earth
Frequency, pp10^8
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
0
2500
5000
7500
10000
12500
15000
17500
Time, Days


PHASE DATA
Oscillator: Earth
Phase, Seconds
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-40.0
0
2500
5000
7500
10000
12500
15000
17500
Time, Days

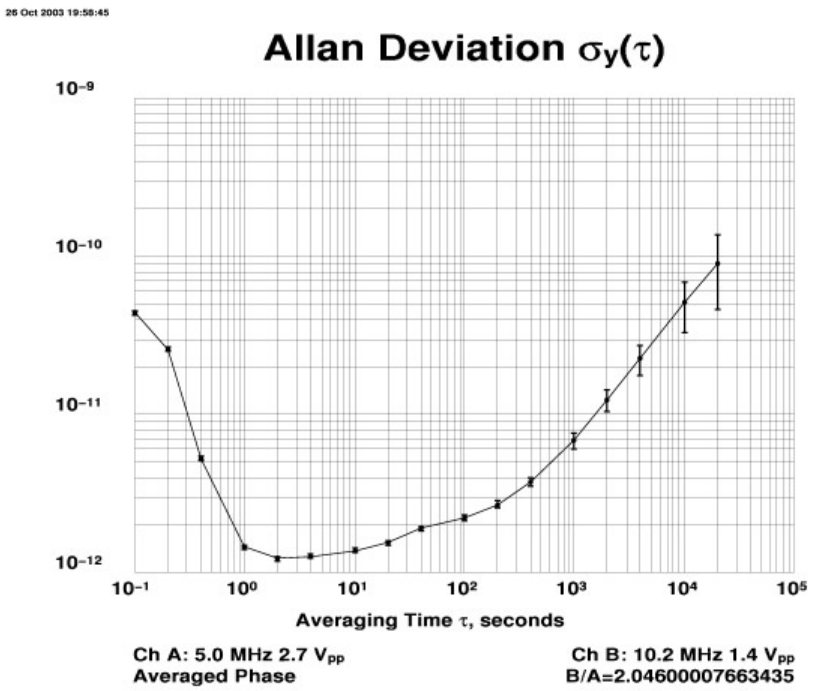

FREQUENCY STABILITY
Oscillator: Earth
Sigma For All Tau
Allan Deviation, σy(τ)
10^-8
3
2
1.5
10^-9
10^4
10^5
10^6
10^7
10^8
10^9
Averaging Time, τ, Seconds

10^{-9} earth

- Earth as a frequency standard
- Suggested improvements:
  - Thoroughly clean, and dry with cloth
  - Remove surrounding gas and water vapor
  - Wait for core to cool before use
  - Re-align axis of rotation (wobbling)
  - Keep away from nearby moon (tides)
  - Keep away from sun (tempco)
  - Re-adjust rate (avoid leap seconds)

10^{-10} ocxo

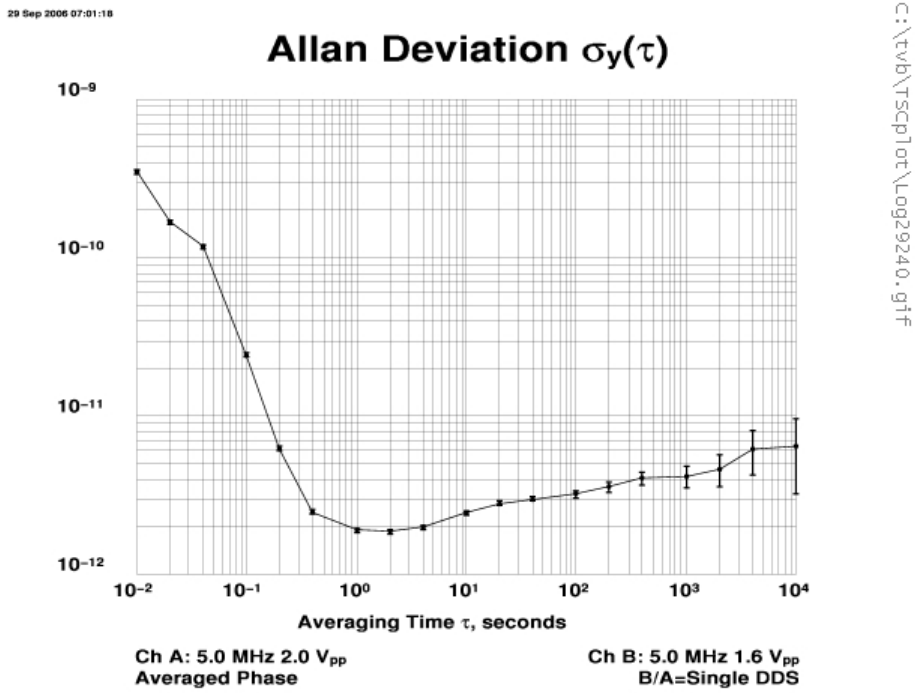
- 0.1 ppb, 100 ps/s, 8.64 μs/d
- 10^{-10}...10^{-13} short
- 5×10^{-10}/d drift



c:\tvb\Tscplot\Log4165.gif

10^{-11} good ocxo

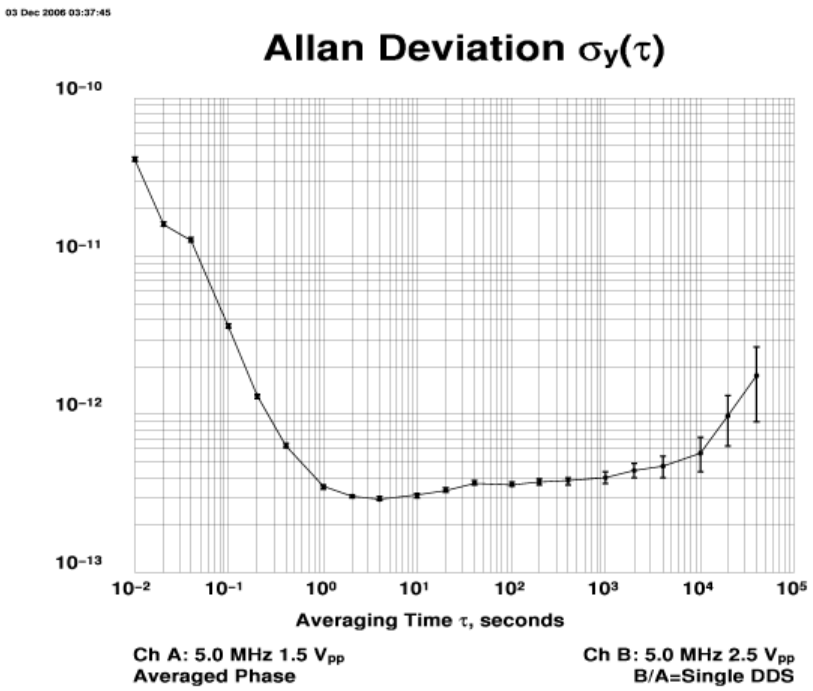
- 0.01 ppb, 10 ps/s, 864 ns/d (~1 μs/d)
- 10^{-11}…10^{-13} short
- ~10^{-11}/d drift



c:\tvb\Tscplot\Log29240.gif

10^{-12} excellent ocxo

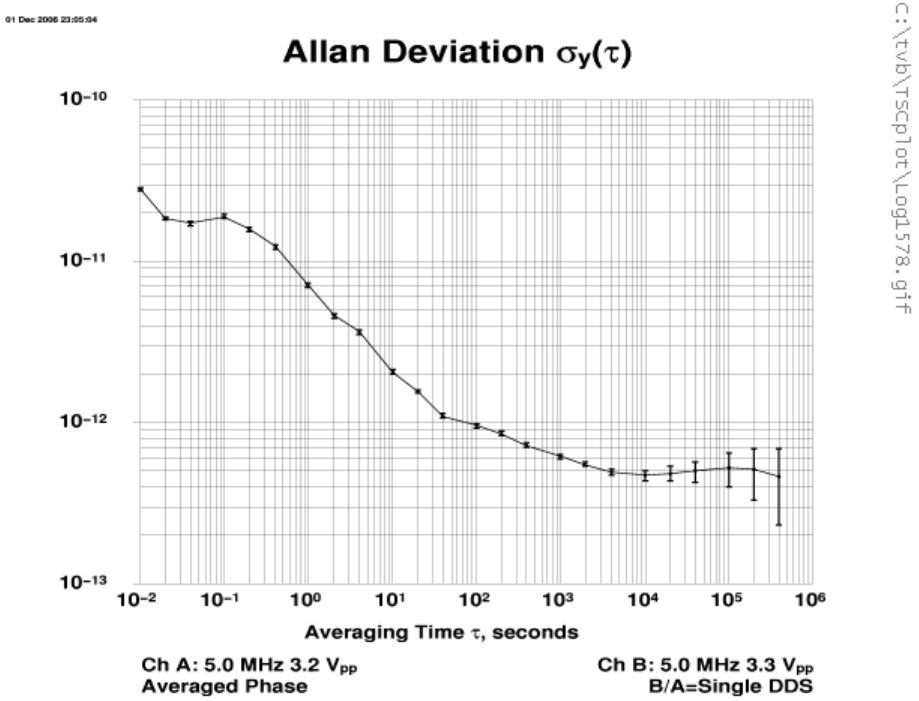
- 1 ppt, 1 ps/s, 86.4 ns/d (~100 ns/d)
- ~10^{-13} short/mid
- ~3×10^{-12}/d drift



c:\tvb\Tscplot\Log29858.gif

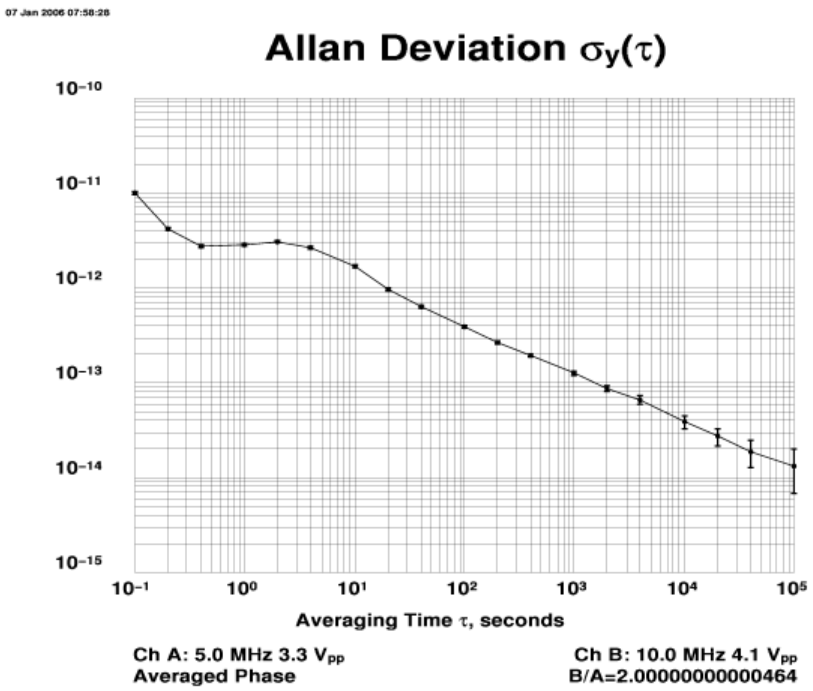
10^{-13} rubidium

- 8.64 ns/d (~10 ns/d)
- ~10^{-13} mid-term
- ~1×10^{-11}/m drift



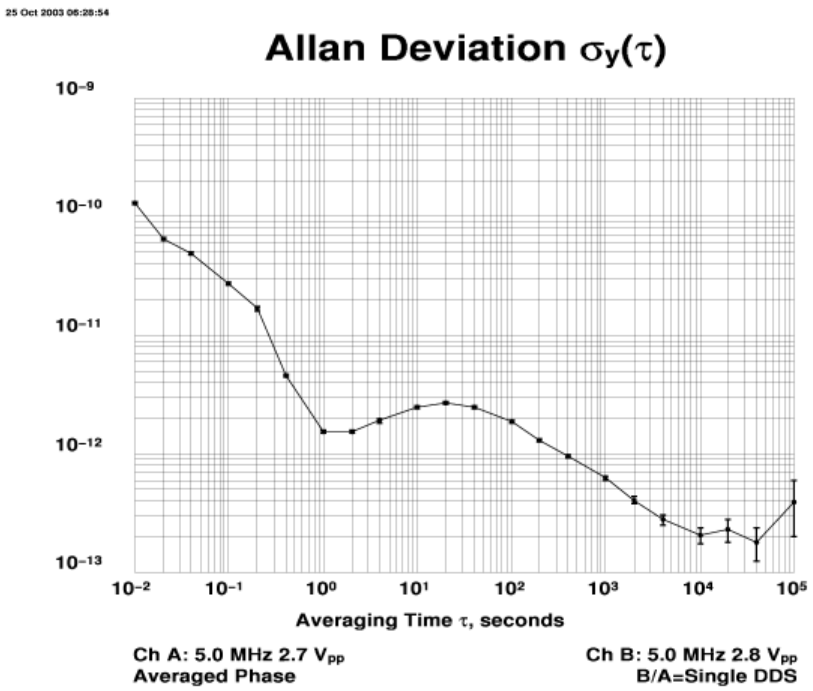
10^{-14} cesium

- 864 ps/d (~1 ns/d)
- ~10^{-13} mid-term
- ~1×10^{-14} @ 1 day



10^{-14} more cesium

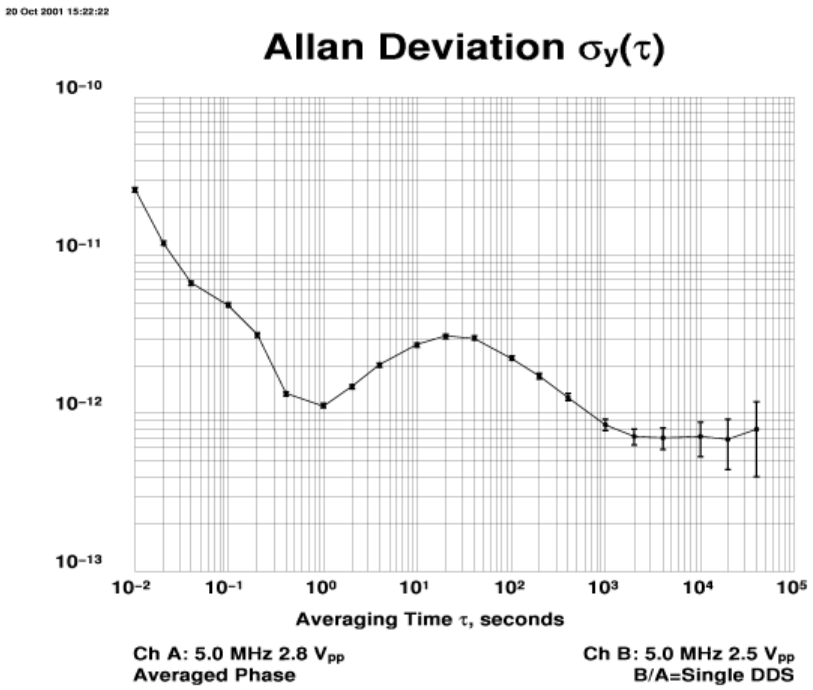
- 10^{-14} not!
- Cesium clocks differ by 2x – 50x
- E.g., old 5060A vs. new 5071A



c:\tvb\Tscplot\Log4143.gif

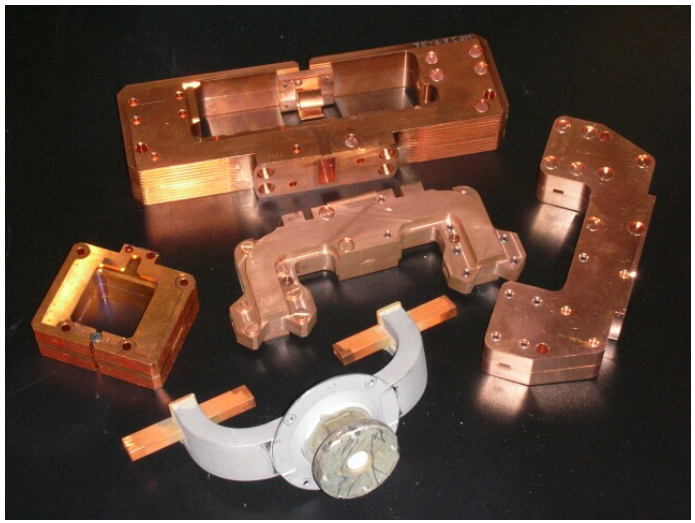
10^{-14} another cesium

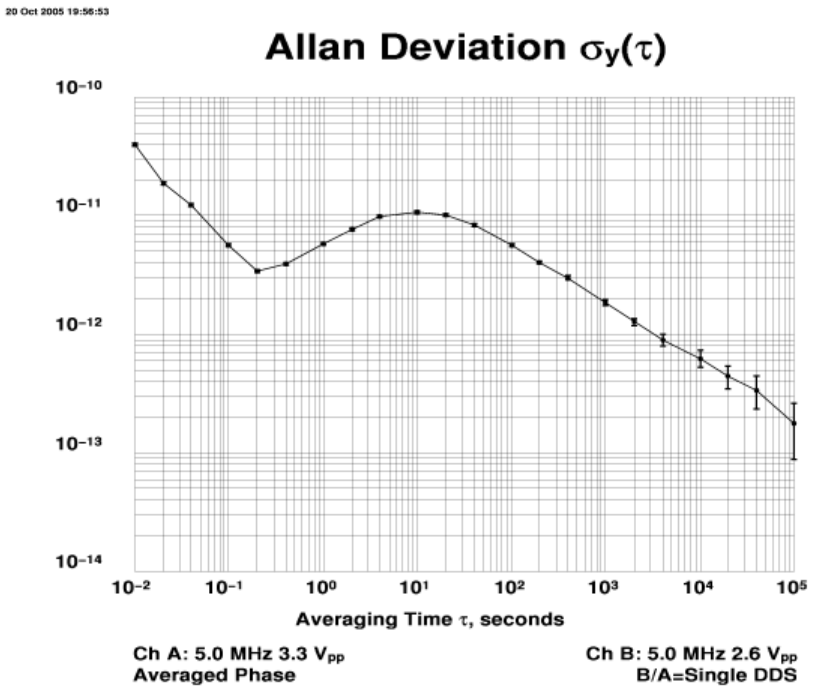
- Not even close to 10^{-14} @ 1 day
- FTS 4010
- Portable Clock
- Old, tired



10^{-14} one more cesium

- About 2×10^{-13} @ 1 day
- FTS 4050
- See variety of Cs beam cavities

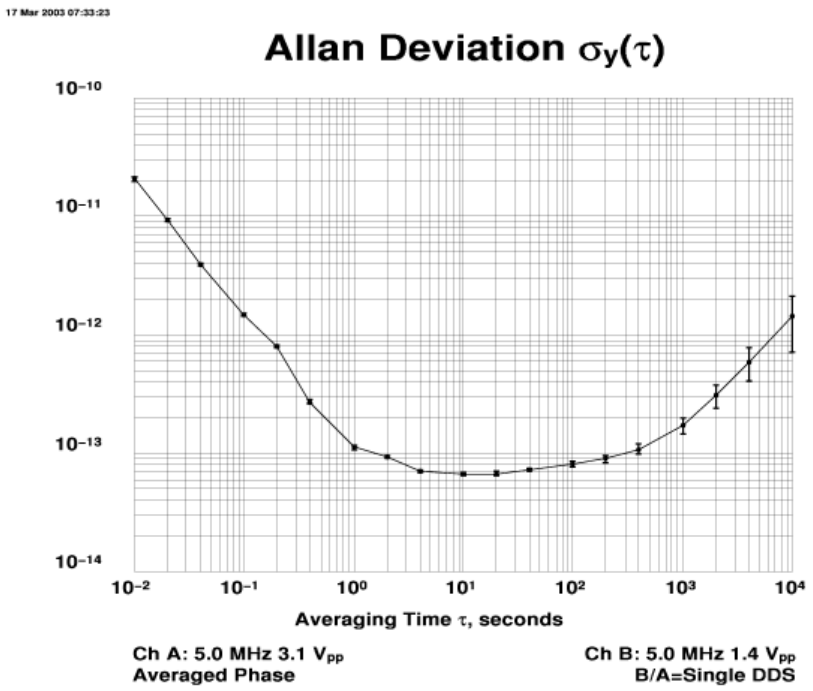




c:\tvb\Tscplot\Log2421.gif

10^{-14} BVA quartz

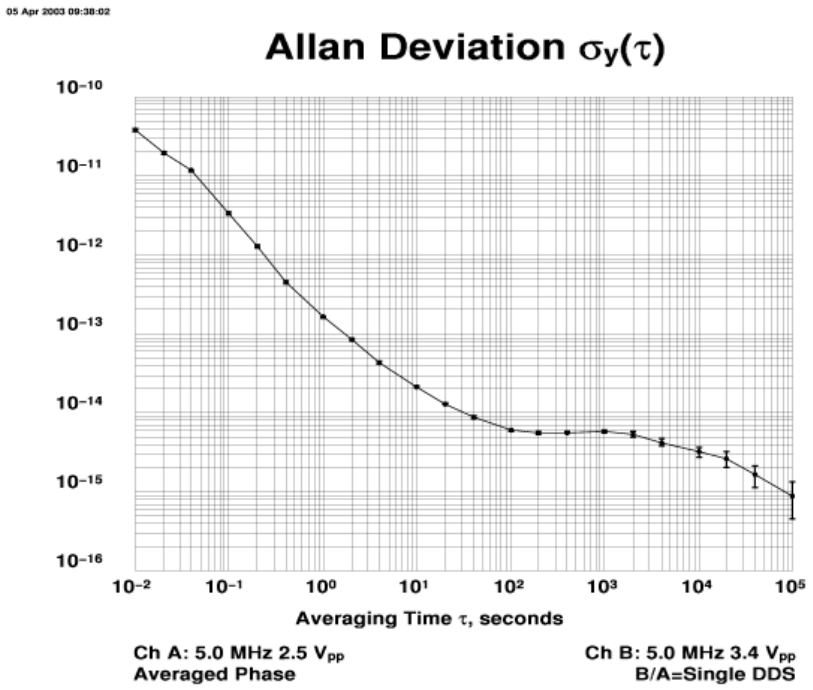
- But can you get to high altitude and measure in 1 to 100 seconds?
- 10^{-13}…10^{-14} short
- 10^{-11}…10^{-12} drift



c:\tvb\TScplot\Log1212.gif

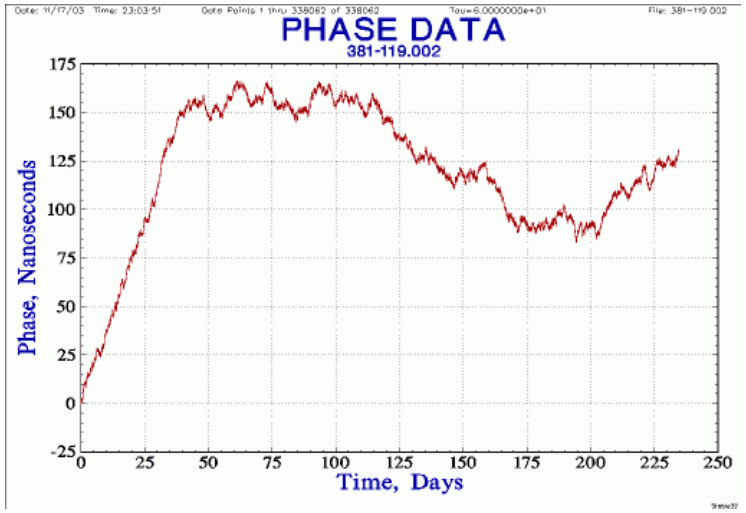
10^{-15} active h-maser

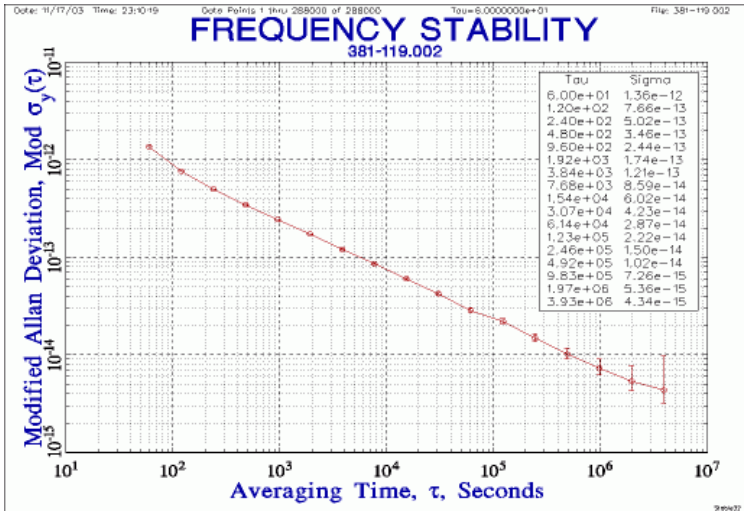
- 86.4 ps/d
- Near 1×10^{-15} @1d
- Cavity auto-tuned



10^{-15} cesium, long-term

- High-perf units
- Pair near 2×10^{-14} at a day
- Floor near 5×10^{-15} in weeks





Powers of Ten – summary

- 10% to 10^{-15} – 15 orders of magnitude